

Total variation image restoration model by adaptively selecting parameter

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Abstract

The choice of the suitable regularization parameter is very important to improve the quality and speed of the image inverse problem. To this end, this paper proposes a new scheme to choose the regularization parameter for the image restoration based on the total variation regularization model. To be more specific, since the proposed model is a nonconvex and nonsmooth optimization problem, we employ the alternating direction method of multipliers (ADMM) to split the original model into several easily solvable subproblems and then the regularization parameter can be efficiently computed by using the meantime Morozov deviation principle. Numerical results show that the proposed model and algorithm are efficient in restoring blurred images that are corrupted by noise.

Keywords: Image Restoration; Total Variation Regularization Model; Parameter Selection; Alternating Direction Method of Multipliers (ADMM); Discrepancy Principle

1 Introduction

The techniques of digital image restoration is an important part of digital image processing [1]. The goal of digital image restoration is to reconstruct an original scene from a degraded observation. This restoration process is essential for many image processing applications. The earliest research on restoration technology can be traced back to the space projects of the United States and the former Soviet Union in the early 1950s and 1960s [2]. And bad imaging environment, equipment vibration, aircraft rotation and other factors make the image have different degrees of degradation. Under the technical background at that time, the noise reduction caused substantial economic losses. Therefore, the application field of image restoration technology is of becoming increasingly

widely used in real life. For example, in the field of remote sensing [3], the image restoration technology can overcome the blur caused by the imaging system relative to the ground movement. And it also can overcome the excessive degradation of image quality caused by atmospheric disturbance, or suppress the coherent noise in radar remote sensing imaging. In addition, image restoration also has important applications in medicine[4], imaging systems such as CT and X-ray need to use image recovery technology to suppress noise in medical images[5]. Therefore, in order to better serve the development of social industries, it is of great practical significance to study the model and algorithm of image restoration technology. In fact, due to the lacking of some prior information,

the restoration problem is considered to be an ill-posed problem. The linear degradation model [6] for image restoration may be expressed as

$$f = Ku + \eta, \quad (1)$$

where $f, u \in \mathbb{R}^{n^2}$ are the observed image and the original image respectively, $K \in \mathbb{R}^{n^2 \times n^2}$ is a fuzzy operator and η is the noise. The early achievements of restoration technology mainly come from some technologies and methods in the field of digital signal processing, among which the simplest and most direct method is inverse filtering technology[7], which is equivalent to solving a least squares problem, namely

$$\min_u \|Ku - f\|_2^2, \quad (2)$$

However, the model (2) is not feasible, because the image recovery problem is often pathological, and the images we observed are often noisy. To this end, we can add the corresponding regular term to the optimization model [8] based on the prior knowledge of the initial image as the following variation energy functional framework

$$\min_u \Phi(u) + \frac{\lambda}{2} \|Ku - f\|_2^2, \quad (3)$$

The tuning parameter $\lambda > 0$ can be used to balance the effect of the data fitting term and the regularization term. Specially, when $\Phi(u) = TV(u) := \|\nabla u\|_1$, the above model becomes a total variation model proposed by Rudin et al.[9]:

$$\min_u TV(u) + \frac{\lambda}{2} \|Ku - f\|_2^2. \quad (4)$$

Currently, there are many ways to solve the model(4). For example, the solution method based on PDE[10, 11] is relatively easy to implement, but requires a lot of iterations to get the accuracy of the solution. Then, famous scholars put forward the Chambolle projection algorithm, which uses dual operators to obtain a minimization problem equivalent to the original problem, which is easy to solve. This algorithm converges quickly, but the optimal iteration step size is not easy to obtain. To solve the above problems, scholars have put forward many improvements, such as FTVD methods [12], by introducing an intermediate variable, the nonlinear term is replaced,

and the penalty term of the replaced term and the intermediate variable is added. In order to make the solution of the problem closer to the original solution, it is necessary to increase the penalty parameter, which makes the condition number of the problem larger and more difficult to solve the problem. At the same time, the numerical calculation will become unstable.

In this paper, we adopt a fast solution method based on the alternating direction multiplier method (ADMM) [13], while considering that image pixel values must be limited within a determined dynamic range $[l, h]$, as the pixel values of an image typically represent the physical energy of the image, so the variable u in the objective function (4) should be non negative. Moreover, the solution obtained by the algorithm mentioned above may contain negative values, so introducing constraints on the target variable is very practical[14]. For example, for an 8-bit image, there is $[l, h] = [0, 255]$, especially when storing or outputting digital images, it is necessary to limit pixel values within $[l, h]$. There are currently many methods to address this issue. In this paper, we adopt the projection method, with the main idea of mapping values less than l to l and values greater than h to h , thereby limiting the pixel values of u and improving the accuracy of the restoration results. Then, for digital image recovery, the model (4) becomes:

$$\min_{u \in \Omega} TV(u) + \frac{\lambda}{2} \|Ku - f\|_2^2, \quad (5)$$

where $\Omega = \{u \in \mathbb{R}^{n^2} / l \leq u \leq h\}$.

The selection of the regularization parameter λ should not only ensure the consistency between the restoration result and the original data, but also ensure that the restoration result is not greatly affected by noise. If the λ value in the model (5) is too large, it is easy to lose a lot of details of the original image [15]. If λ is too small and the denoising effect is poor. Therefore adaptive selection of regularization parameter λ is a challenging problem, but scholars put forward a series of selection methods of regularization parameter. Forecasting risk estimator is unbiased, for example, the (UPRE) method [16], the generalized cross validation (GCV) method [17], L-curve method [18], the difference principle [19], etc. The regularization parameter selection method is

currently developed for a linear Tikhonov regularization or it is truncated singular value decomposition (TSVD) regularization. The TV model of (1), however, the formula for image restoration is nonlinear, so it is difficult to export these evaluation formulas based on the above method.

Although there have been scholars put forward some good TV regularization methods, but these methods only apply to image denoising. Due to motion, docking or atmospheric turbulence and degradation of blurred images, the influence of the point spread function (PSF) in these measures shall not be applied. In this paper [20] puts forward an effective blind image based on the data driven discriminant prior with the fuzzy method, but it is a linear Tikhonov regularization and its regularization is a constant. To efficiently estimate the regularization parameter in the image deblurring model, we use the Morozov deviation principle to select λ . The Morozov deviation principle is to use the residual norm to match the upper bound to select λ , which means that good regular solutions u are in the following set:

$$D = \{u : \|Ku - f\|_2^2 \leq c^2\}, \quad (6)$$

where c is a constant. If the noise variance σ^2 is known, then $c^2 = \tau\sigma^2$. Where τ is the determined number. In general, setting $\tau = 1$ [21]. Under the principle of deviation, the image restoration problem can be expressed as solving a constraint problem as follows:

$$\min_{u \in D} TV(u). \quad (7)$$

Though the Morozov deviation principle is a very effective regularization parameter selection strategy, it is asked to solve an implicit nonlinear equation (Morozov deviation equation). Therefore, an effective solution of Morozov deviation equation is the key to determine the regularization parameters. The classical numerical method for numerically solving the Morozov deviation equation is the Newton method. However, the main disadvantage of the Newton method is that each iteration requires solving a large linear system of equations, which for large scale problems can be prohibitively expensive. To this end, some works [22] propose to solve (7) by transforming the constrained minimizing problem into a non-constrained minimizing problem with the

help of the Lagrangian method (4). It is obvious that problems (4) and (7) are equivalent. If there is a solution to equation (7), it is also a solution to (4) for a specific selected parameter $\lambda \geq 0$ (Lagrangian multiplier). For (7), the supplementary conditions indicate that $u(0) \in D$ or:

$$\|Ku(\lambda) - f\|_2^2 = c^2 \quad (8)$$

for $\lambda > 0$.

If $\lambda = 0$, then the problem (4) is equivalent to the problem (7). Therefore, the solution u is a constant image, which is impossible to occur in real life. Therefore, the deviation principle is trying to find a $\lambda > 0$, and when the problem (7) solution is not a constant image, the problem (8) is satisfied. The advantage of the deviation principle is that when the error level tends to zero, the solution is convergent. that is, the canonical solution tends to the true solution of the original problem. In this paper, we will use the deviation principle to estimate the regularization parameters. In each iteration, we will use the deviation principle to adaptively select regularization parameters. Numerical experiments have also shown that our proposed algorithm performs better.

The rest of this paper is organized as follows. In Section 2, we show an alternating direction multiplier method and give a discrete form of the variable-complete difference of ADMM and solve each sub-problem. In the third part, the appropriate regularization parameters λ are determined. The convex function theorem is given and the specific process of ADMM parameter selection is demonstrated. The validity of the model and the stability of the algorithm are illustrated by the experiments in Section 4. Finally, the important conclusion of this paper is that the algorithm converges quickly and the numerical solution is relatively stable.

2 Alternate direction multiplier method

Alternate Direction Multipliers Method (ADMM) was first proposed by Glowinski and Gabay [23], and it was demonstrated by 2011 by Boyd et al [24]. That can be used to solve large-scale distributed optimization problems. It is widely used in image processing, machine learning, computer vision and other fields. Its basic

idea is to decompose a large global problem into several smaller, easy to solve local subproblems based on the variable separation method, and then use some effective numerical methods. This approach is strongly associated with other solving methods, such as the Lagrange multiplier method.

To the model (4), it is a convex and nonsmooth optimization problem. Then we need to use some operator splitting methods to solve it such as the alternate directional multipliers methods (ADMM) [25], the primal dual methods [26], Douglas-Rachford splitting methods [27], split-Bregman methods [28], and so on. Among these methods, the alternating direction method of multipliers (ADMM) is an algorithm that attempts to solve a convex optimization problem by breaking it into smaller pieces, each of which will be easier to handle. A key step in ADMM is the splitting of variables, and different splitting schemes lead to different algorithms. In order to use the ADMM to solve the model (5), we first introduce an auxiliary variable v and then transform it into the following constraint problem

$$\begin{cases} \min_{u \in \Omega, v} \|v\|_1 + \frac{\lambda}{2} \|Ku - f\|_2^2 \\ \text{s.t.} \begin{cases} v = \nabla u, \\ w = u, \\ z = u. \end{cases} \end{cases} \quad (9)$$

In order to use the ADMM, we need to introduce the augmentation Lagrangian function

$$\begin{aligned} & L_A(u, v, w, z; \alpha, \gamma, \mu) \\ = & \|v\|_2 - \langle \alpha, v - \nabla u \rangle + \frac{\beta_1}{2} \|v - \nabla u\|_2^2 \\ & - \langle \gamma, w - u \rangle + \frac{\beta_2}{2} \|w - u\|_2^2 \\ & + \frac{\lambda}{2} \|Kz - f\|_2^2 - \langle \mu, z - u \rangle + \frac{\beta_3}{2} \|z - u\|_2^2, \end{aligned}$$

where $\beta_1, \beta_2, \beta_3 > 0$ are the penalty parameters, α and γ, μ are the Lagrange multipliers. For the above-mentioned Lagrangian function can be solved by an alternating iterative methods:

$$\begin{pmatrix} v^{k+1} \\ w^{k+1} \end{pmatrix} = \arg \min_{w \in \Omega, v} L_A(u^k, v, w, z^k; \alpha^k, \gamma^k, \mu^k), \quad (10)$$

$$\begin{aligned} u^{k+1} &= \arg \min_u L_A(u, v^{k+1}, w^{k+1}, z^k; \alpha^k, \gamma^k, \mu^k), \\ z^{k+1} &= \arg \min_z L_A(u^{k+1}, v^{k+1}, w^{k+1}, z; \alpha^k, \gamma^k, \mu^k), \end{aligned} \quad (11)$$

$$(12)$$

$$\begin{pmatrix} \alpha^{k+1} \\ \gamma^{k+1} \\ \mu^{k+1} \end{pmatrix} = \begin{pmatrix} \alpha^k - \beta_1(v^{k+1} - \nabla u^{k+1}) \\ \gamma^k - \beta_2(w^{k+1} - u^{k+1}) \\ \mu^k - \beta_3(z^{k+1} - u^{k+1}) \end{pmatrix}. \quad (13)$$

Now we discuss how to solve the problems (10)-(12).

- To the problem (10), it is a convex but nonsmooth and can be written as

$$v^{k+1} = \arg \min_v \|v\|_2 + \frac{\beta_1}{2} \left\| v - \nabla u^k + \frac{\alpha^k}{\beta_1} \right\|_2^2.$$

This is the classic soft shrinkage problem and then its closed-form solution can be obtained by

$$v^{k+1} = \max \left\{ \left\| \nabla u^k + \frac{1}{\beta_1} \alpha^k \right\|_2 - \frac{1}{\beta_1}, 0 \right\} \cdot \frac{\nabla u^k + \frac{1}{\beta_1} \alpha^k}{\left\| \nabla u^k + \frac{1}{\beta_1} \alpha^k \right\|_2}. \quad (14)$$

- To the problem (10), it can be rewritten as

$$w^{k+1} = \arg \min_{w \in \Omega} \frac{\beta_2}{2} \|w - u^k\|_2^2 - \langle \gamma^k, w - u^k \rangle$$

This is a convex and constraint optimization problem. We can use the projection gradient method to solve it as

$$\begin{aligned} w^{k+1} &= \mathcal{P}_\Omega \left(u^k + \frac{\gamma^k}{\beta_2} \right) \\ &= \min \left(h, \max \left(u^k + \frac{\gamma^k}{\beta_2}, l \right) \right) \end{aligned} \quad (15)$$

- For the problem (11), it can be reorganized as

$$\begin{aligned} u^{k+1} &= \arg \min_u \frac{\beta_1}{2} \|v^{k+1} - \nabla u\|_2^2 - \langle \alpha^k, v^{k+1} - \nabla u \rangle \\ &\quad - \langle \gamma^k, w^{k+1} - u \rangle + \frac{\beta_2}{2} \|w^k - u\|_2^2 \\ &\quad - \langle \mu^k, z^k - u \rangle + \frac{\beta_3}{2} \|z^k - u\|_2^2. \end{aligned}$$

This is a least square problem and its optimization condition is

$$\left(\Delta + \frac{\beta_2 + \beta_3}{\beta_1} I\right) u = \text{div} A^k + \frac{\beta_2 \beta_3}{\beta_1} B^k, \quad (16)$$

where Δ is the Laplace Operator and div is the adjoint operator of the gradient operator ∇ . In addition, here $A^k = v^{k+1} - \frac{\alpha^k}{\beta_1}$ and $B^k = \frac{w^{k+1}}{\beta_3} + \frac{z^k}{\beta_2} - \frac{\gamma^k + \mu^k}{\beta_2 \beta_3}$.

To the equation (16), if we assume that the gradient operator uses the periodic boundary conditions when it is discretized, then Δ is a block cyclic matrix [29, 30], which can be diagonalized to [31] by the discrete Fourier transform. With this assumption, we can get u^{k+1} by

$$u^{k+1} = \mathcal{F}^{-1} \left(\frac{\mathcal{F}(\text{div}) \mathcal{F}(A^k) + \frac{\beta_2 \beta_3}{\beta_1} \mathcal{F}(B^k)}{\mathcal{F}(\Delta) + \frac{\beta_2 + \beta_3}{\beta_1} \mathcal{F}(I)} \right), \quad (17)$$

where $\mathcal{F}$ is the fast Fourier transform and $\mathcal{F}^{-1}$ denotes the inverse of $\mathcal{F}$.

- To the problem (12), it can be reorganized as

$$z^{k+1} = \arg \min_z \frac{\lambda}{2} \|Kz - f\|_2^2 - \langle \mu^k, z - u^{k+1} \rangle + \frac{\beta_3}{2} \|z - u^{k+1}\|_2^2.$$

This problem is the classic least square problem and we can get the solution by

$$z^{k+1} = (\lambda K^T K + \beta_3 I)^{-1} (\lambda K^T f + M_k), \quad (18)$$

where

$$M_k = \mu^k + \beta_3 u^{k+1}.$$

In fact, to the image deblurring problem, the operator K is assumed to be the point spread function and then K is a circulant matrix to its discretization. Then we use the FFT to solve the equation (18) by

$$z^{k+1} = \mathcal{F}^{-1} \left(\frac{\mathcal{F}(\lambda K^T f + M_k)}{\mathcal{F}(\lambda K^T K + \beta_3 I)} \right), \quad (19)$$

Using the ADMM to solve the problem (5) can be summarized as follows.

Algorithm I ADMM solving the model (5)

- Enter the $f, k > 0, \beta_1, \beta_2, \beta_3 > 0$, Initialize $u = u^0, z = z^0$ and $\alpha = \alpha^0, \gamma = \gamma^0, \mu = \mu^0$.

- ① Based on (14), updating out v^{k+1} ;
- ② Based on (15), updating w^{k+1} ;
- ③ Based on (16), updating u^{k+1} ;
- ④ Based on (18), updating gain z^{k+1} ;
- ⑤ Based on (13), updating $\alpha^{k+1}, \gamma^{k+1}, \mu^{k+1}$;

- If the stopping can not satisfied, updating above iteration ①—⑤. Otherwise, outputting the restored image u^{k+1} .
-

3 Adaptive selection of the regularization parameter

In each step iteration of the algorithm, the deviation principle is used to determine the regularization parameter λ . To get the appropriate λ , our approach is to iterate, where adaptively adjusting λ so that recovering the image in the set D is the feasible. The relevant formula is as follows:

$$z^{k+1}(\lambda_{k+1}) = (\lambda_{k+1} K^T K + \beta_3 I)^{-1} (\lambda_{k+1} K^T f + M_k), \quad (20)$$

Therefore, we can obtain the error e_{k+1} of the solution in the following form:

$$\begin{aligned} e_{k+1} &= K z^{k+1}(\lambda_{k+1}) - f \\ &= K (\lambda_{k+1} K^T K + \beta_3 I)^{-1} (\lambda_{k+1} K^T f + M_k) - f, \end{aligned}$$

Use the equation $K(tK^T K + I)^{-1} = (tKK^T + I)^{-1}K$, get out:

$$e_{k+1} = (\lambda_{k+1} K K^T + \beta_3 I)^{-1} (K M_k - f), \quad (21)$$

where, we define that

$$P(\lambda, M) \equiv \|(\lambda K K^T + \beta_3 I)^{-1} (K M - f)\|_2^2. \quad (22)$$

We have the following theorem showing that when $\|K M_k - f\|_2^2 > c^2$ there must be a unique regularization parameter λ_{k+1} , so $\|K z^{k+1}(\lambda_{k+1}) - f\|_2^2 = c^2$.

Theorem 1 Make $t = KM - f, P(\lambda, M)$ as shown in (22). Then $P(\lambda, M)$ is strictly about λ monotonically decreasing positive convex functions, and have

$$P(\lambda, M) = b^2, \quad (23)$$

There is a unique solution $\mu > 0$, valid for any b that satisfies $\|t_0\|_2^2 \leq b^2 \leq \|t\|_2^2$. Here, t_0 is by t in KK^T Orthogonal projection on a zero space.

The theorem is proved by directly computing the first and second derivatives of $P(\lambda, M)$ about λ . Strict convexity of P ensures λ uniqueness. For Theorem 1, we know that there is a unique solution to $P(\lambda, M_k) = c^2$ when $\|KM_k - f\|_2^2 > c^2$, is also $M_k \notin D$ which means we can find the unique $\lambda_{k+1} > 0$, make $\|Kz^{k+1}(\lambda_{k+1}) - f\|_2^2 = c^2$. When $M_k \in D$, we can select $\lambda_{k+1} = 0$, make $z^{k+1} = M_k \in D$. To sum up, we must be a unique $\lambda_{k+1} \geq 0$ making $z^{k+1} \in D$. Notice that (22) is nonlinear, and we can solve it with the Newtonian method. Therefore, the ADMM of parameters is selected according to the deviation principle. The specific process is as follows:

Algorithm II (ADMM based on deviation principle Solve the constrained TV-L2 model(9))

- Enter $f, k > 0, \beta_1, \beta_2, \beta_3 > 0$, Initialize $u = u^0, z = z^0$ and $\alpha = \alpha^0, \gamma = \gamma^0, \mu = \mu^0$.
 - Based on (14), updating out v^{k+1} ;
 - Based on (15), updating w^{k+1} ;
 - Based on (16), updating u^{k+1} ;
 - Have $M_k = \mu^k + \beta_3 u^{k+1}$;
 - If $M_k \in D$;
 - Then $\lambda_{k+1} = 0$;
 - If $M_k \notin D$;
 - Solve $P(\lambda_{k+1}, M_k) = c^2$;
 - Have $z^{k+1}(\lambda_{k+1}) = (\lambda_{k+1}K^TK + \beta_3I)^{-1}(\lambda_{k+1}K^Tf + M_k)$;
 - By means of (13) to correct it $\alpha^{k+1}, \gamma^{k+1}, \mu^{k+1}$;
 - Determine whether the termination condition is met, if satisfied, the image z^{k+1} , is output, otherwise, the second step is returned.
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4 Numerical experimentation

In this part, we will illustrate the effectiveness of the model (9) and the stability of the ADMM algorithm. In this paper, the algorithm ends when the selection iteration reaches 200 steps, and the algorithm results of the fixed regularization parameters are compared with the adaptive selected regularization parameters to verify the effectiveness of the proposed algorithm

and model. Since the algorithm contains multiple parameters, to select the parameters efficiently, we first save the degraded graph as a mat format and then debug the relevant parameters to optimize the experimental results. The experimental platform in the paper is adopted Matlab 9.6 on PC (Intel Core i5 4590, 3.30GHz), where the fuzzy images are obtained using the Matlab function $H = \text{fspecial}('gaussian', Hsize, Sigma)$ and $Blur = \text{imfilter}(I, H, 'conv')$. In the experiments, we measure the observed image quality and the recovered image quality by using PSNR (Peak Signal-to-Noise Ratio) and SSIM (Structural Similarity Index Measurement).

To compare our method with other methods, we select Aight natural images to test as shown in Fig.1. Consider the gray-scale image Cameraman and Lena as the test image, where the parameters used by the fuzzy image are $Hsize=3$, The coefficients select 0.1,0.05,0.01,respectively. Figure.2 and Figure.3 show the recovery result of the grayscale image Cameraman and Lena, which shows that the fixed parameter of image restoration are superior to the adaptive parameter method when the coefficient is equal to 0.1, however, the model recovery effect of adaptive selection of regularization parameters is better than that of fixed regularization parameters when the coefficient is equal to 0.01 and 0.05, which can well reflect the detailed characteristics of the image. So our model has a very good recovery effect(See Table 1 and Table 2 for details).

We also restore Pepper of 256×256 pixel images degraded by random noise. The middle and bottom of Fig.4 are the corresponding restored images with the method of fixed parameters and adaptive selected regularization parameters respectively. Fig.4 shows that the image quality of our method is more effective than that of fixed parameters. The comparisons of our method and fixed parameters are shown. We observe the PSNR and SSIM of our method are higher than those of the Fixed parameters(See Table 1 and Table 2 for details).

Recovery results by different models for Mandrill are as shown in Fig.5. The first row is a degraded image and corrupted by the random noise. The second row is the corresponding restored images with fixed parameters. The third row is corresponding restored images by the adaptive selected regularization parameters used in the



Figure 1 Original images with the same size. From left to right:(a).Cameraman(256×256) (b).Lena(256×256) (c).Pepper(256×256) (d).Mandrill(256×256) (e).Boat(256×256) (f).BaJie(256×256) (g).Clown(256×256) (h).FigPage(256×256)



Figure 2 Cameraman recovery rendering of the diagram. The three columns are shown from left to right the COEFFICIENT select 0.1, 0.05, 0.01. First row behavior degradation image; model recovery result of second behavior regularization parameter $\lambda = 25$; third behavior adaptation selects the model recovery result of the regularization parameter

model. Fig.5 shows that the image recovered from the model adapted for selecting the regularization parameters has a more obvious contrast.



Figure 3 Recovery renderings of the *Lena* diagram. The three columns are shown from left to right the COEFFICIENT select 0.1, 0.05, 0.01. First row behavior degradation image; a model complex of the second behavior regularization parameter $\lambda = 25$ original result; the third behavior adaptation selects the model recovery result of the regularization parameters *Lena*

We restore Boat of 256×256 pixel images degraded by the random noise. Fig.6 shows that the image quality of adaptive selected regularization parameters is more effective than that

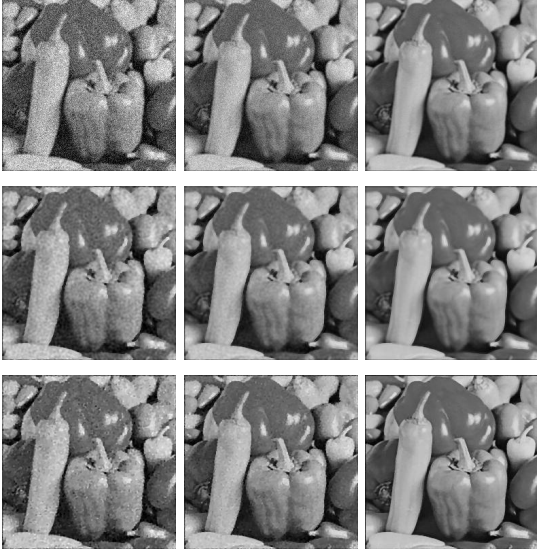


Figure 4 Top:Pepper degraded image and corrupted by the random noise.The three columns are shown from left to right the COEFFICIENT select 0.1, 0.05,0.01.Ground floor:It is the original images of Pepper.Middle:Corresponding restored images by fixed parameters.bottom:Corresponding restored images by adaptive selected regularization parameters.

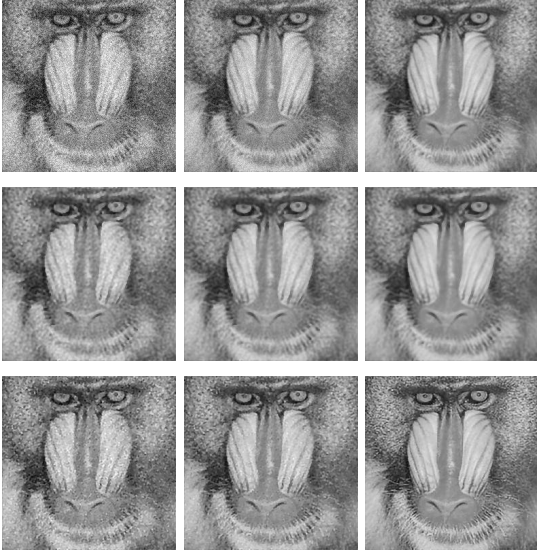


Figure 5 Recovery renderings of the Mandrill diagram.The three columns are shown from left to right the COEFFICIENT select 0.1,0.05,0.01.A first behavior degradation image;a model complex of the second behavior regularization parameter $\lambda = 25$ Original result;the third behavior adaptation selects the model recovery result of the regularization parameters Mandrill

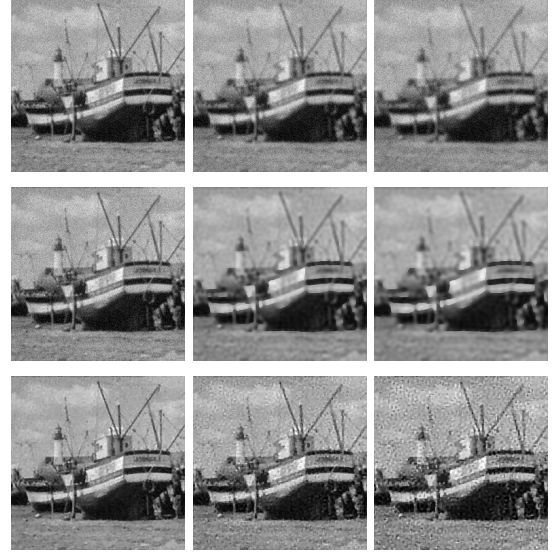


Figure 6 Boat recovery rendering of the diagram.The three columns are shown from left to right the COEFFICIENT=0.05 and Hsize select 3,5,7.First row behavior degradation image;model recovery result of second behavior regularization parameter $\lambda = 25$;third behavior adaptation selects the model recovery result of the regularization parameter.It is clear from the above that adaptive selected regularization parameters is more effective in image quality.On the one hand,the noise estimation strategy is adopted to ensure the rationality of the adaptive parameter selection,on the other hand,the adaptive parameter selection mechanism ensures the adaptability of the algorithm under different noise,To increase the anti-morbid problem.

of fixed parameters. We observe the PSNR and SSIM of our method are higher than those of the fixed parameters. Consider the gray-scale image BaJie as a test image, where the parameters used by the fuzzy image are respectively the COEFFICIENT=0.05 and Hsize select 3,5,7.Below for the BaJie grayscale. Recovery of the image and making the experimental renderings,see Figure 6.

Consider the classic Clown image. Clown images have more special than BaJie grayscale images. Fig. 7 shows the recovery result of the grayscale image BaJie, which shows that the ADMM has a very good recovery effect. However, the model recovery effect of adaptive selection of regularization parameter is better than that of fixed regularization parameter, which can well reflect the detailed characteristics of the image. We restore Clown of 256×256 pixel images degraded by the random noise. The middle and bottom of Fig.8 are the corresponding restored



Figure 7 BaJie recovery rendering of the diagram. The three columns are shown from left to right the COEFFICIENT=0.05 and Hsize select 3,5,7. First row behavior degradation image; model recovery result of second behavior regularization parameter $\lambda = 25$; third behavior adaptation selects the model recovery result of the regularization parameter

images with the method of fixed parameters and adaptive selected regularization parameters respectively. Fig.8 shows the image quality of our method is more effective than that of fixed parameters. Our method also can get a better image recovery performance. We observe the PSNR and SSIM of our method higher than those of the fixed parameters.

Recovery results by different models for Fig-Page are as shown in Fig.9. The first row is degraded image and corrupted by the random noise. The second row is corresponding restored images by fixed parameters. The third row is corresponding restored images by adaptive selected regularization parameters used in the models. Fig.9 shows that the image recovered from the model adapted for selecting the regularization parameters has a more obvious contrast.

Through all experimental results, it can be found that our proposed algorithm can better recover the detail texture information of the image, and the boundary contour is also better processed. It also can to obtain the description of the image more accurately (See Table 3 and Table 4 for details).



Figure 8 Clown recovery rendering of the diagram. The three columns are shown from left to right the COEFFICIENT=0.05 and Hsize select 3,5,7. First row behavior degradation image; model recovery result of second behavior regularization parameter $\lambda = 25$; third behavior adaptation selects the model recovery result of the regularization parameter

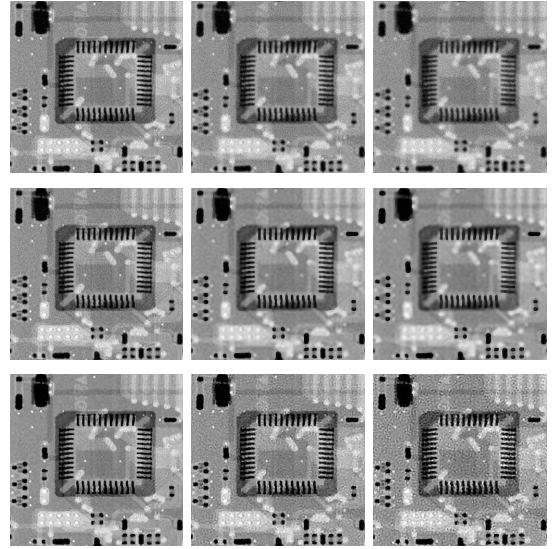


Figure 9 FigPage recovery rendering of the diagram. The three columns are shown from left to right the COEFFICIENT=0.05 and Hsize select 3,5,7. First row behavior degradation image; model recovery result of second behavior regularization parameter $\lambda = 25$; third behavior adaptation selects the model recovery result of the regularization parameter

Table 1 This is compared with fixed parameters and adaptive selected regularization parameters choice of the two methods about 4 images(Cameraman, Lena,Pepper,Mandrill) with Hsize=3 for PSNR values in the experiment

where */* respectively represent:fixed parameters/adaptive selected regularization parameters

Image	PSNR ¹		
	the COEFFICIENT=0.1	the COEFFICIENT=0.05	the COEFFICIENT=0.01
Cameraman	24.32/23.46	25.61/26.45	26.11/30.97
Lena	25.85/24.51	27.87/28.11	28.72/33.10
Pepper	24.46/23.86	25.86/26.86	32.69/32.80
Mandrill	19.71/19.44	20.05/20.66	20.16/23.00

Note:The experimental images from the 1st to 4th rows are respectively Cameraman ,Lena ,Pepper ,Mandrill.From left to right,they are PSNR values of Hsize=3,THE COEFFICIENT select 0.1,0.05,0.01.

Table 2 This is compared with fixed parameters and adaptive selected regularization parameters choice of the two methods about 4 images(Cameraman, Lena,Pepper,Mandrill) with Hsize=3 for SSIM values in the experiment

where */* respectively represent:fixed parameters/adaptive selected regularization parameters (Hsize=3).

Image	SSIM ²		
	THE COEFFICIENT=0.1	the COEFFICIENT=0.05	the COEFFICIENT=0.01
Cameraman	0.61/0.51	0.75/0.71	0.84/0.92
Lena	0.69/0.62	0.82/0.79	0.88/0.93
Pepper	0.68/0.62	0.81/0.77	0.94/0.94
Mandrill	0.43/0.44	0.47/0.56	0.49/0.76

Note:The experimental images from the 1st to 4th rows are respectively Cameraman ,Lena ,Pepper ,Mandrill.From left to right,they are SSIM values of Hsize=3,THE COEFFICIENT select 0.1,0.05,0.01.

Table 3 This is compared with fixed parameters and adaptive selected regularization parameters choice of the two methods about 4 images(Boat,BaJie, Clown,FigPage) with the COEFFICIENT=0.05 for PSNR values in the experiment

where */* respectively represent:fixed parameters/adaptive selected regularization parameters

Image	PSNR ¹		
	HSIZE=3	HSIZE=5	HSIZE=7
Boat	25.70/26.45	23.95/23.32	23.26/21.15
BaJie	26.53/27.60	24.31/22.89	23.33/19.11
Clown	26.82/26.85	25.19/23.75	24.25/20.45
Figpage	28.2228.61	25.80/24.52	24.48/21.89

Note:The experimental images from the 5th to 8th rows are Boat,BaJie ,Clown ,FigPage. From left to right, they are PSNR values of the COEFFICIENT=0.05 and Hsize select 3,5,7.

5 Conclusion

For the non-differentiability of the TV regular term in the image recovery TV model, we propose a solution method based on the alternating direction multiplier method. The algorithm has the following advantages:

- (1) It has the fast iterative convergence speed and good results with a little iteration;
- (2) The numerical solution is relatively stable.

For the selection of the regularization parameters, we use the deviation principle to select the regularization parameters, in the iteration process, self-adaptive. The regularization parameters should be adjusted so that the parameters are optimal and the image recovery effect is better.

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Data Availability : All data, models, and code generated or used during the study appear in the submitted article.

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Table 4 This is compared with fixed parameters and adaptive selected regularization parameters choice of the two methods about 4 images(Boat,BaJie, Clown,FigPage) with the COEFFICIENT=0.05 and Hsize select 3,5,7 for SSIM values in the experiment

where */* respectively represent:fixed parameters/adaptive selected regularization parameters

Image	SSIM ²		
	H SIZE=3	H SIZE=5	H SIZE=7
Boat	0.73/0.74	0.65/0.55	0.61/0.43
BaJie	0.83/0.80	0.80/0.60	0.77/0.45
Clown	0.80/0.77	0.75/0.60	0.72/0.45
Figpage	0.80/0.80	0.73/0.61	0.70/0.48

Note:The experimental images from the 5th to 8th rows are respectively Boat,BaJie,Clown,FigPage.From left to right,they are SSIM values of the COEFFICIENT=0.05 and Hsize select 3,5,7.

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